Representing forecast errors at convective scale

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Int.Symposium on DA – Münich - 2014

METEO FRANCE
Toujours un temps d'avance

Why representing forecast errors ?

For Data Assimilation :

•Background error covariances **B**

To make better use of forecasts

•Error variances : predictability of meteorological events

•Probability : risk of occurrence of strong events

Difficulties :

•Physical complexity : Error’s PDFs vary in space and in time

•Numerical complexity : lack of information to estimate the full PDFs

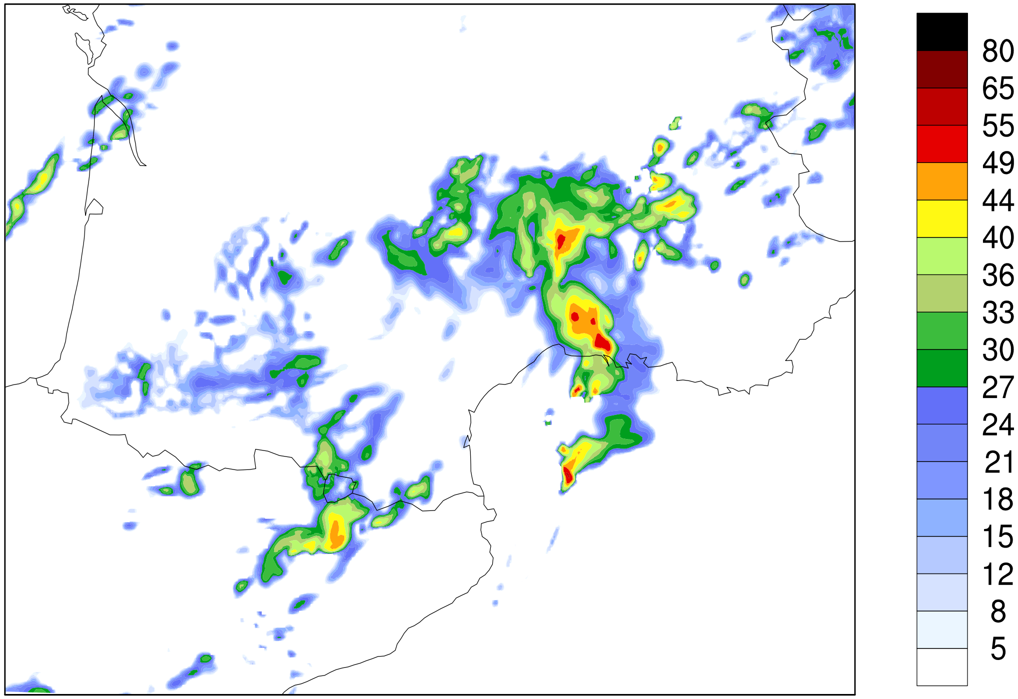
$\Rightarrow$ Need to model covariances and/or use of ensembles

Context: NWP at convective scale

Non-hydrostatic models (in the 1-3 km horizontal resolution range) allow realistic representation of convection, clouds, precipitation, turbulence, surface interactions

Specific features:

- Need coupling models to provide LBCs and surface conditions
- Observations linked to clouds and precipitation can be considered (e.g radars)
- Analyses must be performed frequently
- Forecasts are very expensive in computation time!!



Radar reflectivity simulated by AROME

Outlines

- Specific features compared to global scale
- **B** modelling
  - Climatological formulation
  - Adding some flow dependencies
  - Considering hydrometeors
- **B** totally or partly deduced from ensembles
  - principles
  - role of localization
- Towards optimal filtering of forecast error parameters from ensembles
- **Conclusions**

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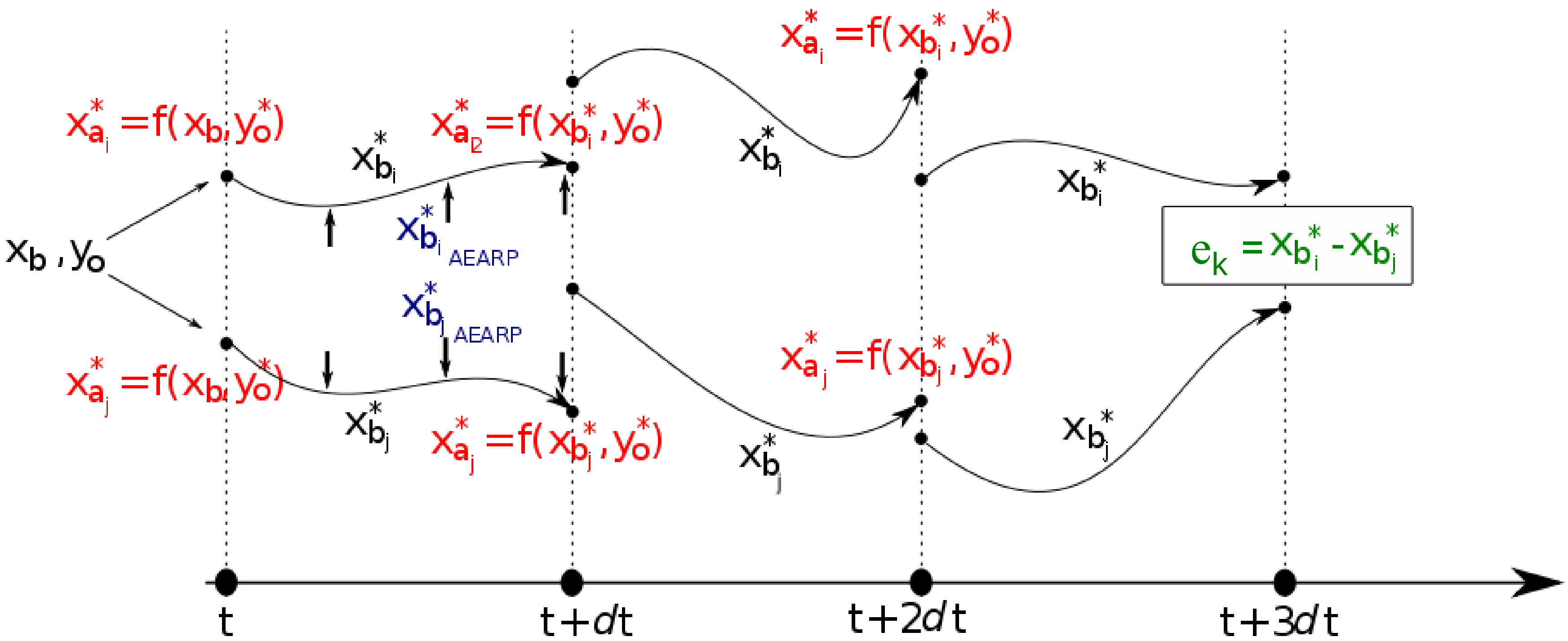
Specific features compared to Global scale

(Ménétrier, Montmerle, Berre and Michel, QJRMS 2014)

1. **Use of an EDA based on AROME with 90 members** to produce a reference database of backgrounds : AEARO-90



Explicit perturbation of obs: $y^*_o = y_o + e_o \quad (e_o \sim N(0, \sigma_o^2))$

Exicit perturbation of LBCs using AEARP members

Implicit perturbation of background: $x^*_b = M(x^*_a) + (e_m)$

Fisher 2003 ; Kucukkaraca and Fisher (2006); Berre et al 2006

Specific features compared to Global scale

2. Approximate B using *N* backgrounds and their mean :

$$\widetilde{B}_{ij} = \frac{1}{N-1}\sum_{p=1}^{N}\left(x_{i,p}^{b} - \langle x_i^b \rangle\right)\left(x_{j,p}^{b} - \langle x_j^b \rangle\right)$$

3. Split $\widetilde{\mathbf{B}}$ in variances / correlations: $\widetilde{\mathbf{B}} = \widetilde{\mathbf{V}}^{1/2}\widetilde{\mathbf{C}}\widetilde{\mathbf{V}}^{\mathrm{T}/2}$

- Correlations can be approximated locally using the tensor of the Local Correlation Hessian (LCH, Weaver and Mirouze (2012)) :

$$\boldsymbol{H} = -\nabla\nabla^{\mathrm{T}} c(\mathbf{r})|_{\mathbf{r}=0}$$

- $\boldsymbol{H}$ is computed using the covariances of normalized perturbation derivatives (Michel 2012)
- Local correlation lengths are then deduced in the direction of the eigen vectors of $\boldsymbol{H}$ using its eigen values:

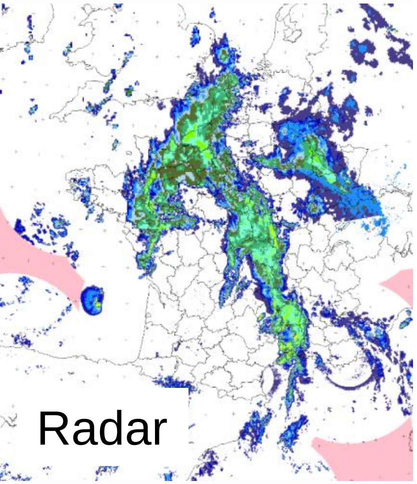
$$L_{\gamma}^{\mathrm{b}} = \lambda_{\gamma}^{-1/2}$$

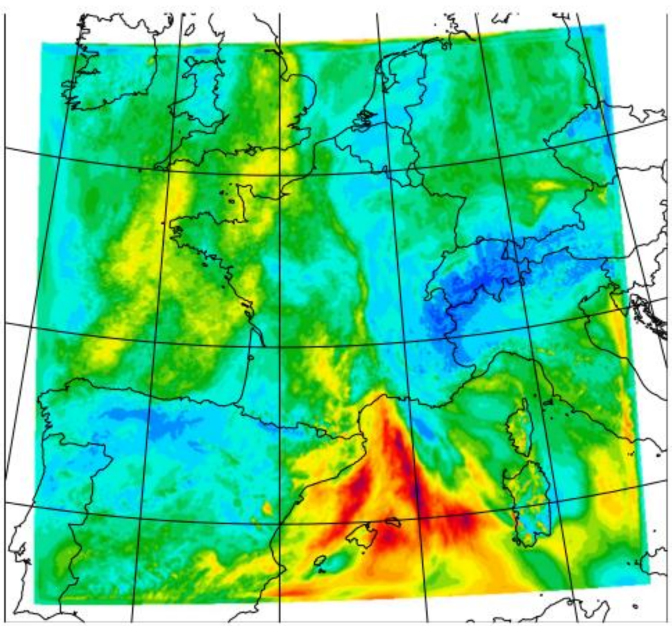
Specific features compared to Global scale

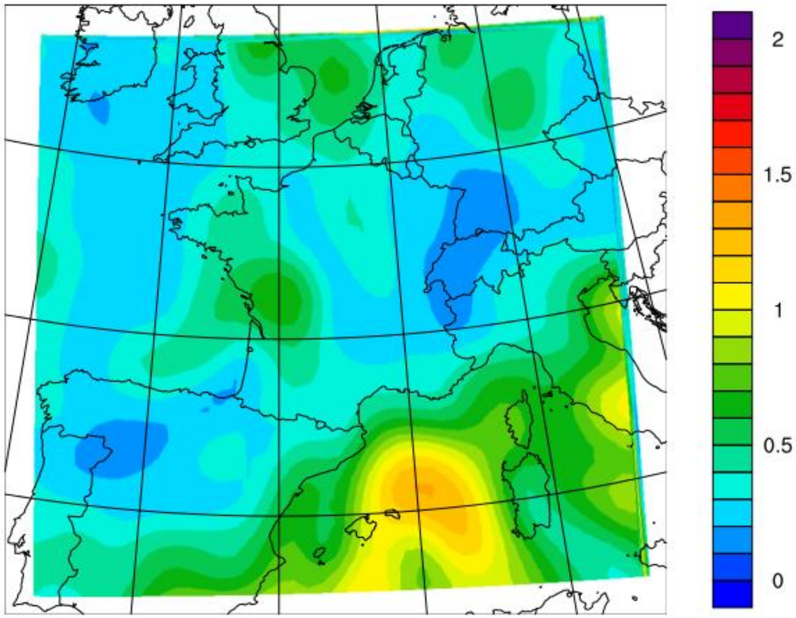
Raw background error variances for q at 945 hPa :

AEARO 90
conv. scale

AEARP 90
global scale





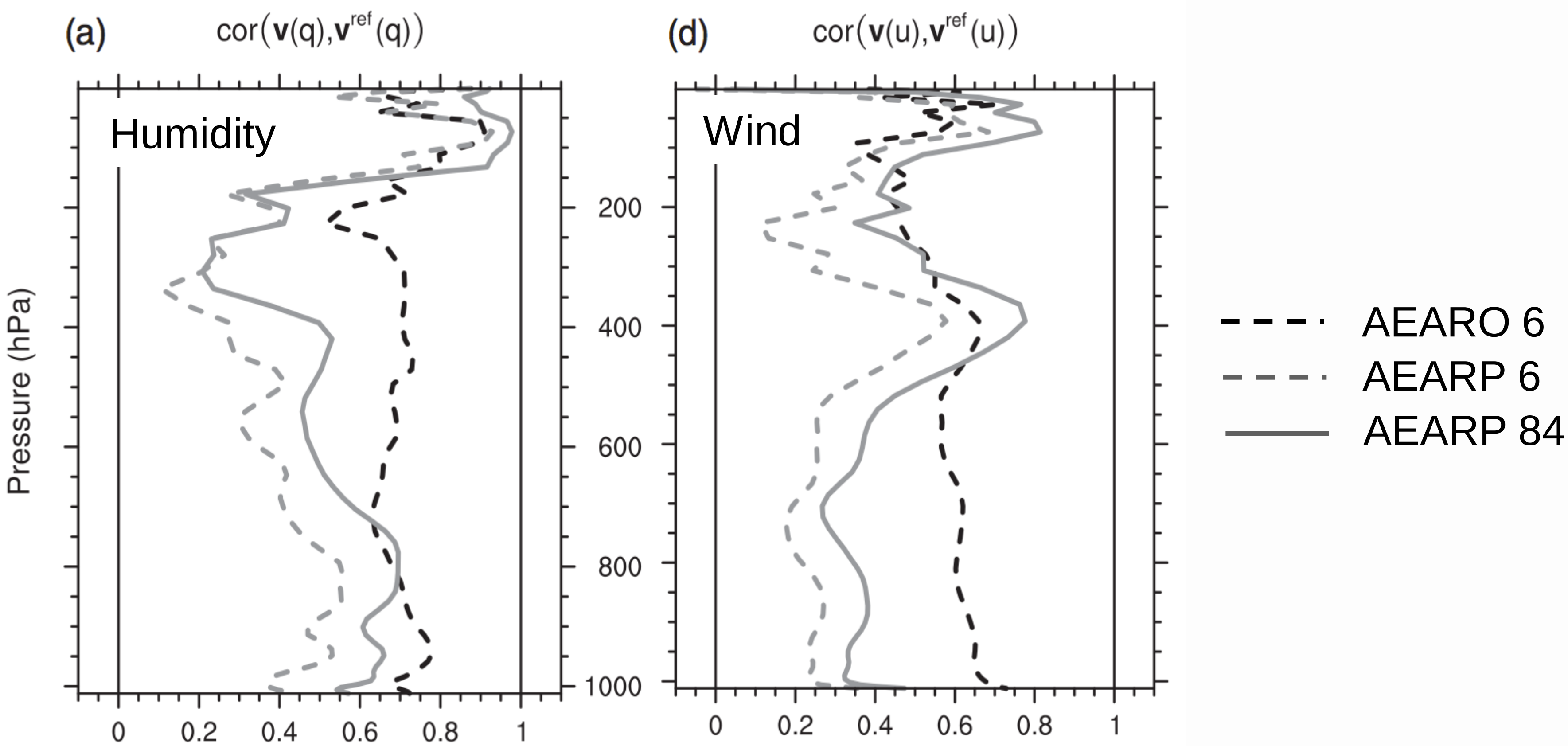


Stronger spatial gradients and larger values due to :

- the resolution difference and the underlying different physical parameterizations
- the assimilation of different observations (e.g radars)

Specific features compared to Global scale

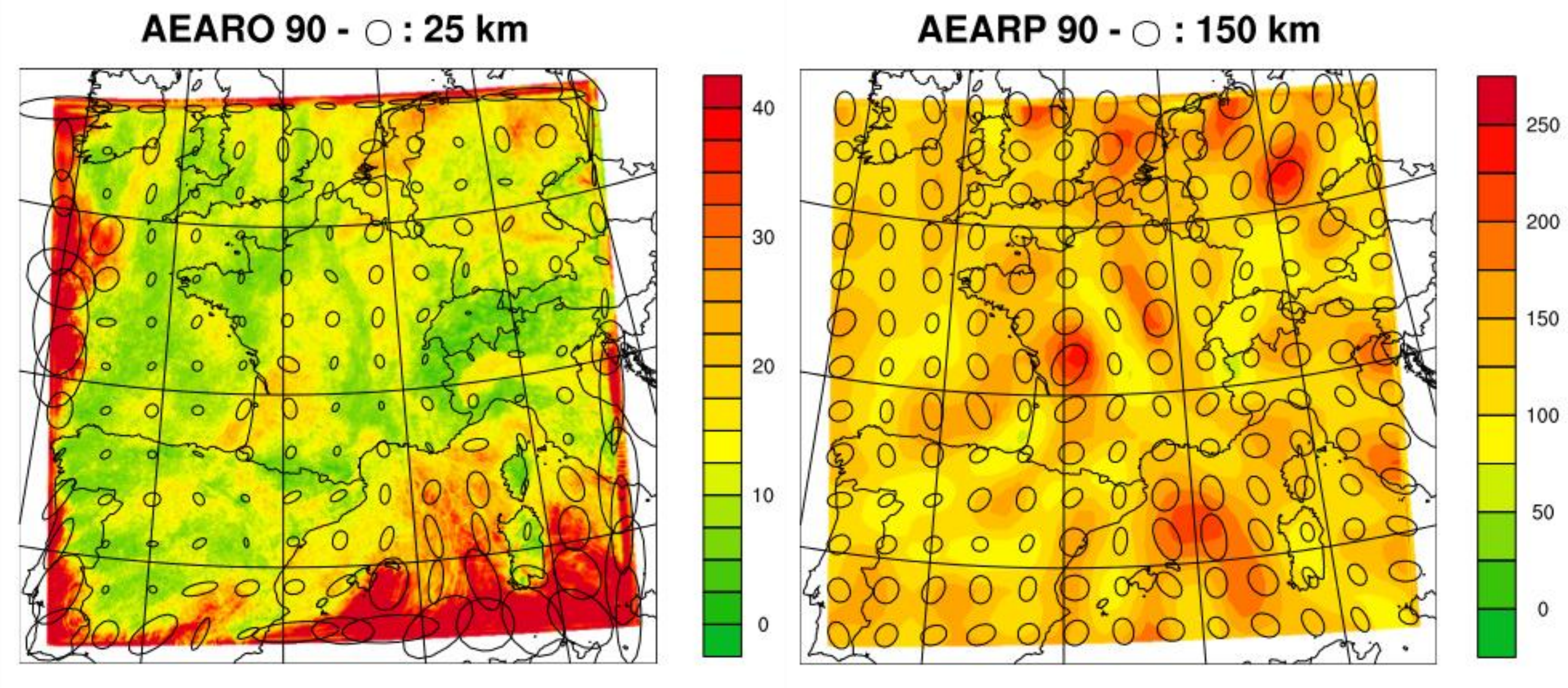
Spatial correlation of raw background-error variances with respect to the AEARO-84



⇒ Downscaling from global models seems not adapted for mid-latitude applications

Specific features compared to Global scale

Total L_b and ellipses of the LCH tensor
(=> correlation shapes around the origin) for q at 945 hPa



- Much shorter length-scales, much more anisotropic structures
- Small values over mountains and in precipitations
- LS perturbations advected inside the domain due to coupling

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B modelling

An operational NWP system at convective scale :

- **Uses sequential variational DA with frequent assimilation/forecast steps** to benefit from observations with high temporal resolutions

- is commonly based on an incremental formulation with CVT transform (Courtier et al., 1994)

$$dx = \mathbf{B}_C^{1/2}\, c$$

- uses a sequence of sparse operators to model $\mathbf{B_c}$, that can not be expressed at full rank (~$(10^8)^2$)

⇒ The challenge is to capture in $\mathbf{B_c}^{1/2}$ the known important features of $\mathbf{B}$

B modelling

Typical structure of $B_c^{1/2}$:

(Derber and Bouttier (1999))

$$\mathbf{B}_C^{1/2} = \mathbf{K}_P \mathbf{B}_S^{1/2}$$

- $\mathbf{K_p}$: **Balance operators** (or parameter transform) that decorrelate multivariate relationships

⇒ Typically transforms to variables which are assumed to be uncorrelated, using analytical operators and regression coefficients

- $\mathbf{B_S}^{1/2}$: **Spatial transforms** that decorrelate univariate unbalanced variables + variance scaling. Horizontal correlations are **generally homogeneous and isotropic**

⇒ Such formulation allows to get balanced analyzed fields

⇒ Those operators are calibrated using database of forecasts (e.g NMC method, EDA) to get climatological static values

(More details can be found in Bannister (2008))

B modelling

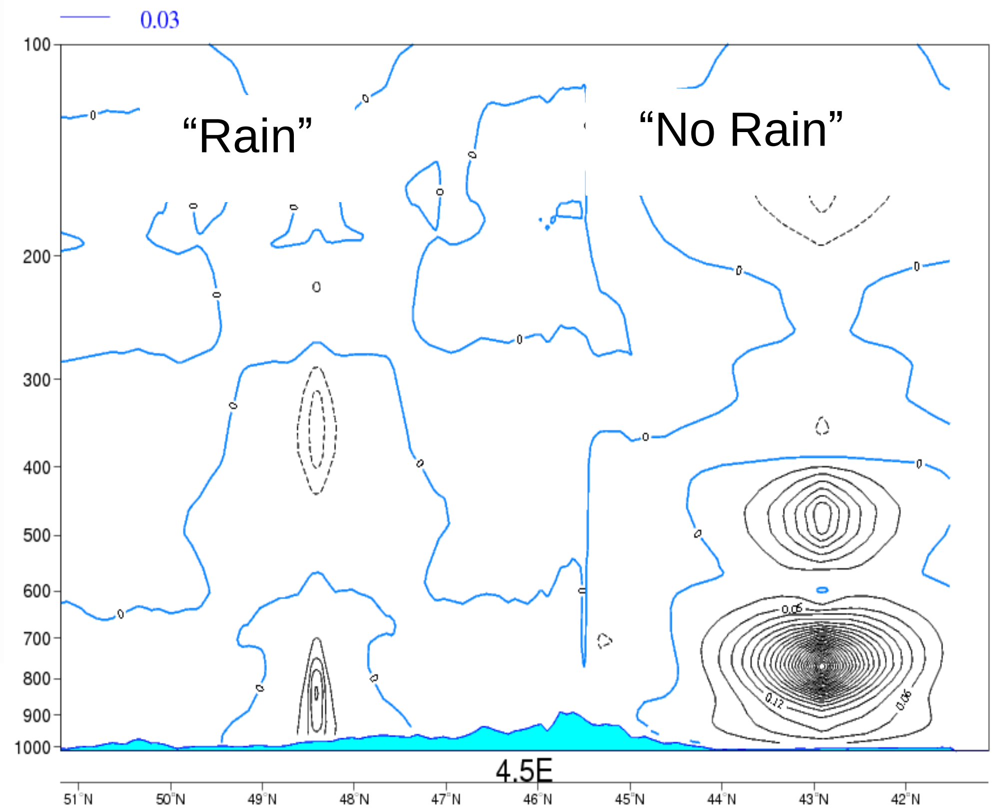
Known limitations of B_c

For LAM, strong dependencies to weather regimes (Brousseau et al., 2011) and to meteorological phenomena that are under-represented in the ensemble

⇒ **Often high impact weather!**

Example 1 : rain

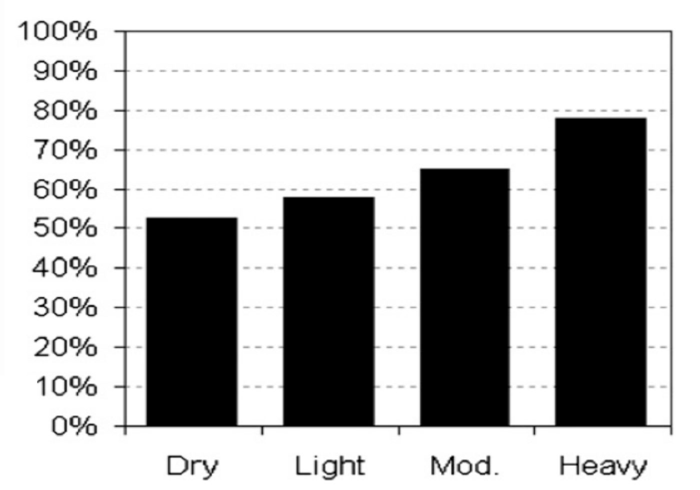
Humidity increments obtained with a $\mathbf{B_c}$ modeled using only ensembles of precipitating profiles and applied in rainy areas using a heterogeneous 3DVar (Montmerle and Berre 2010)



B modelling

Normalized deviation from linear geostrophical balance for different types of rain

(Carron and Fillion (2010))



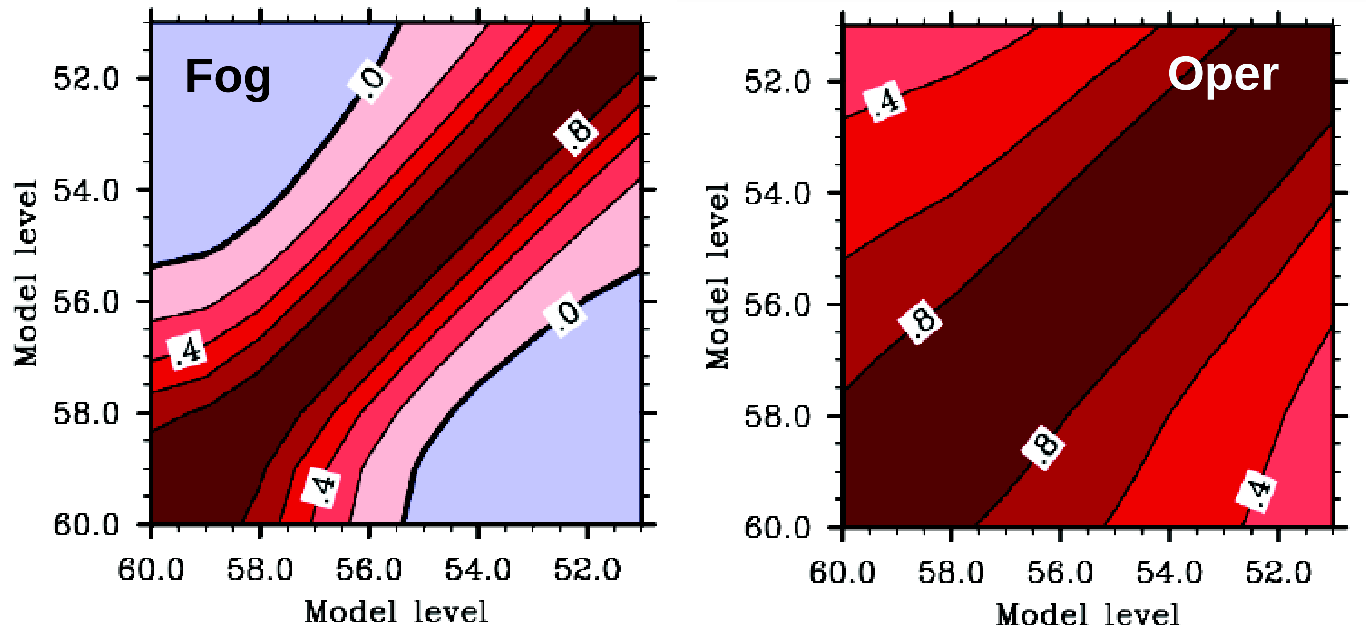
Also, deviation from hydrostatic balance
(Vetra-Carvalho et al. 2012)

Example 2 : fog

Vertical auto-correlations for T

(zoom in the first 500m)

Ménétrier and Montmerle (2011)



B modelling

Adding some flow dependencies in Bc

1. In the balance operator

•For the larger scales (Fisher, 2003): NLBE, Quasi-Geostrophic omega and continuity equations

•A diabatic forcing of balanced vertical motion, as diagnosed by Pagé et al. (2007) could (hardly) be introduced

2. In horizontal covariances (using large ensembles)

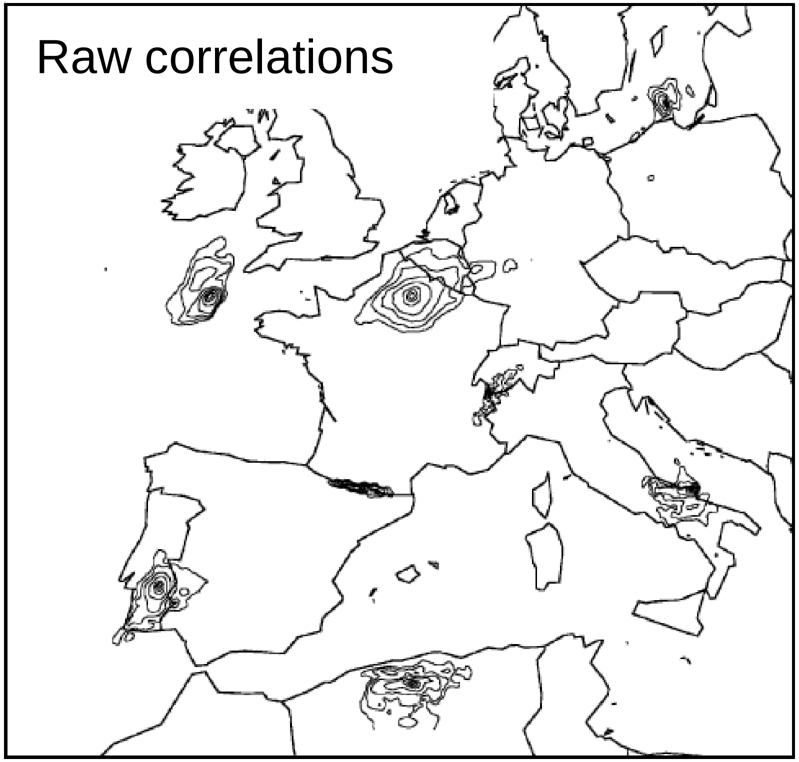
- Wavelet formulation allows to model simultaneously scale and position-dependent aspects of covariances (Fisher, 2003)
- Streching of covariances in recursive filters (Purser et al., 2003b)
- Isotropic correlations computed in a distorted grid (Michel 2012)

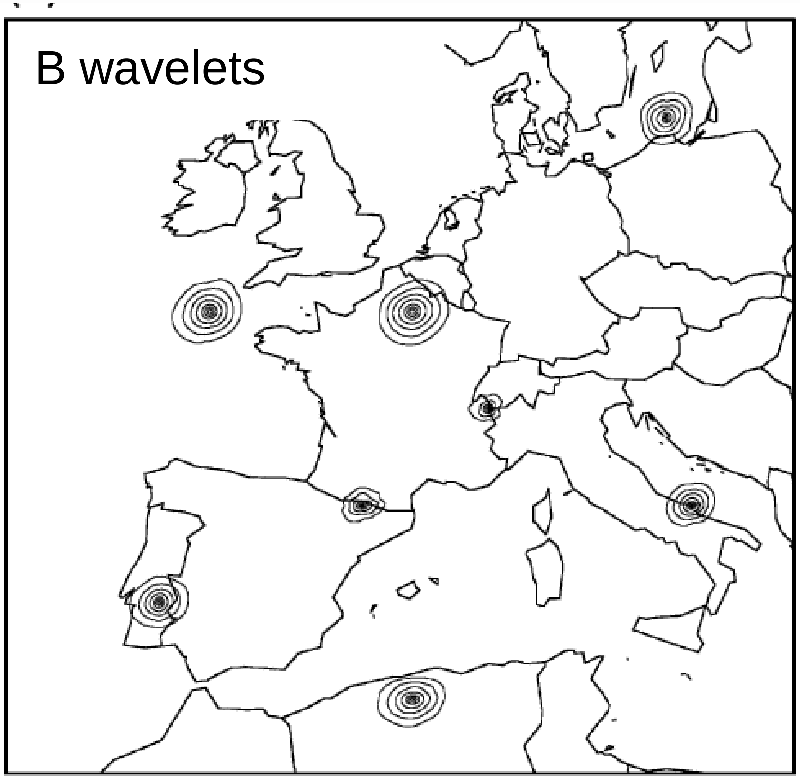
3. In vertical covariances

- Deformation of vertical correlations based on the static stability of the guess (Piccolo and Cullen (2012))

B modelling

Mesoscale Horizontal correlations based on wavelets
(Deckmyn and Berre (2005))



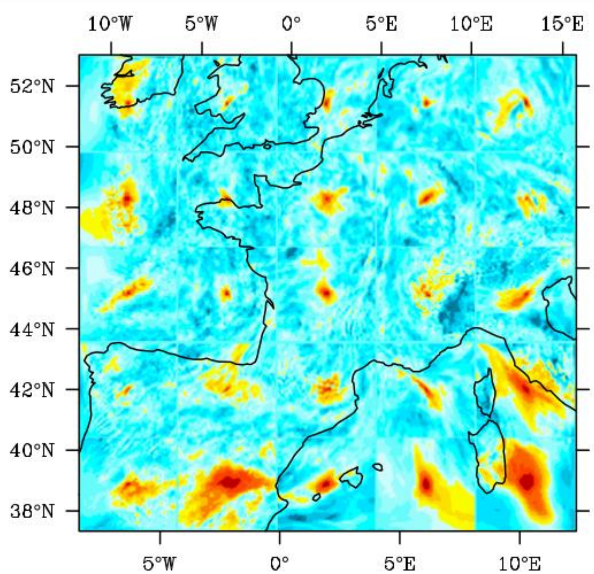


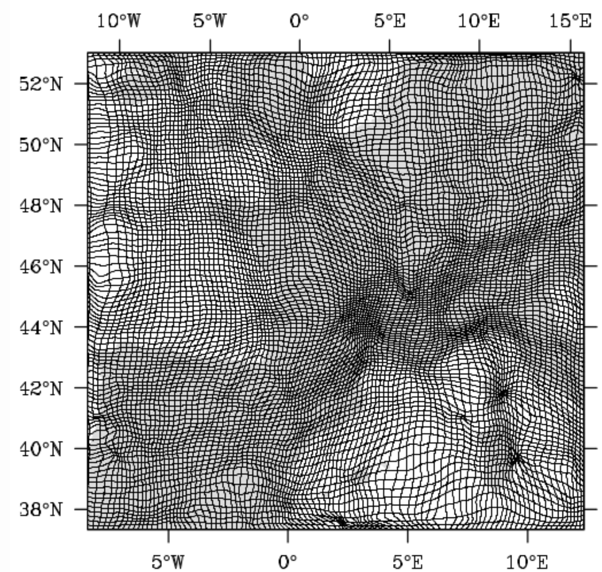
⇒ Shorter length-scales over orography, larger over seas

B modelling

Grid deformation for horizontal error correlations (Michel 2012)

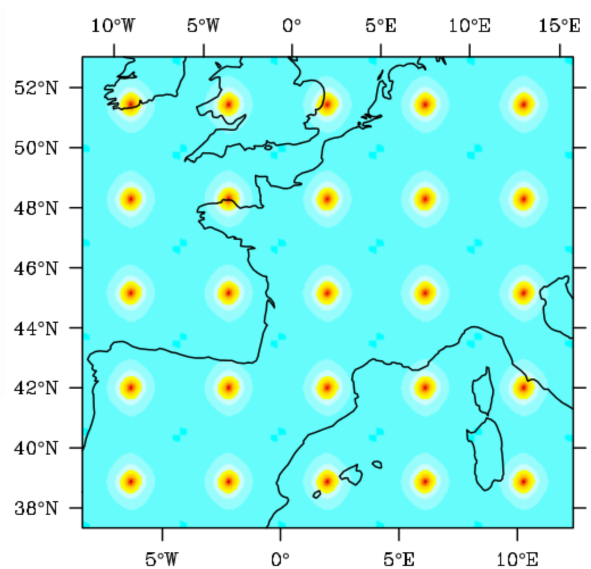
Raw

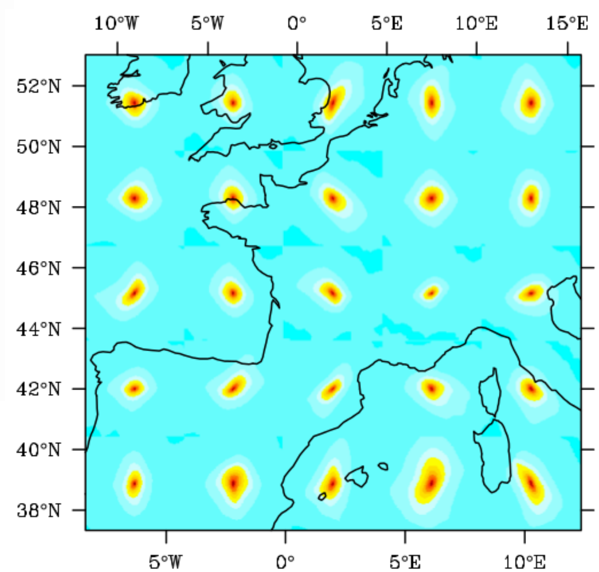




Distorted grid (where correlations are homogeneous and isotropic)

Modeled diagonal spectral

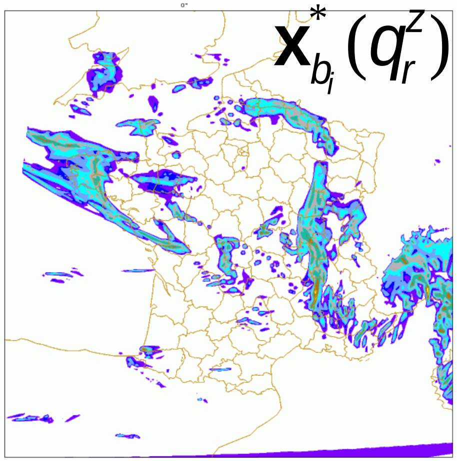




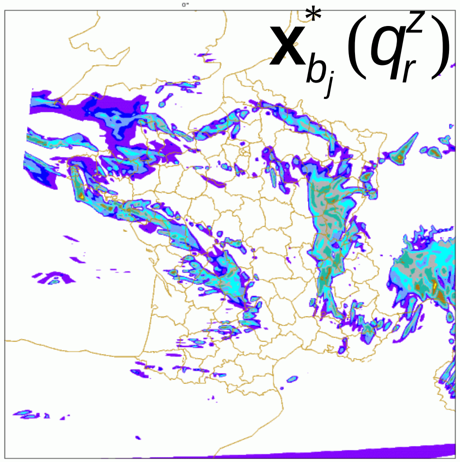
Back to regular grid

B modelling for hydrometeors

1. Use of geographical masks in B modeling (as in Montmerle and Berre (2010))



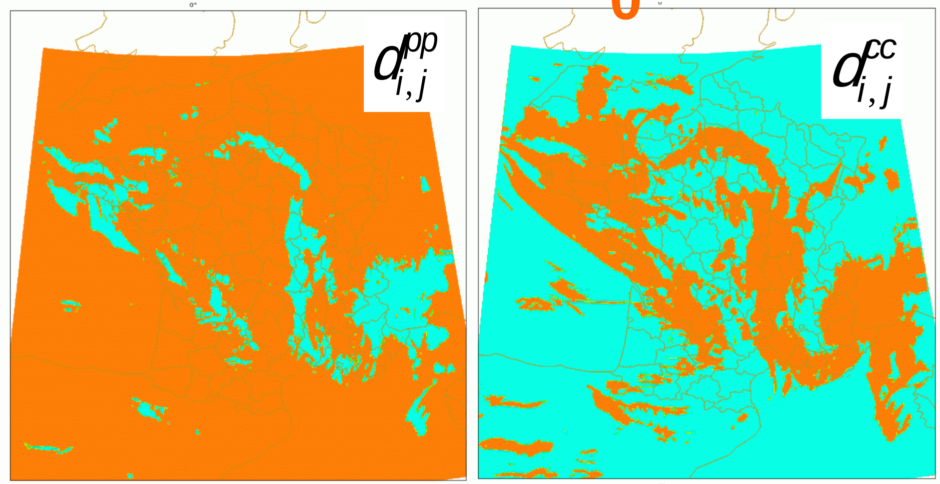
$\mathbf{x}^*_{b_i}(q_r^z)$



$\mathbf{x}^*_{b_j}(q_r^z)$

Example for precipitation

Binary masks: 1 0



$d^{pp}_{i,j}$ $d^{cc}_{i,j}$

rain/rain *clear / clear*

$$e_{f_{ij}} = \mathbf{x}^*_{b_i} - \mathbf{x}^*_{b_j} \approx \left[G d^{pp}_{ij} \right] e_{f_{ij}} + \left[G d^{cc}_{ij} \right] e_{f_{ij}} + \left[G d^{cp}_{ij} \right] e_{f_{ij}}$$

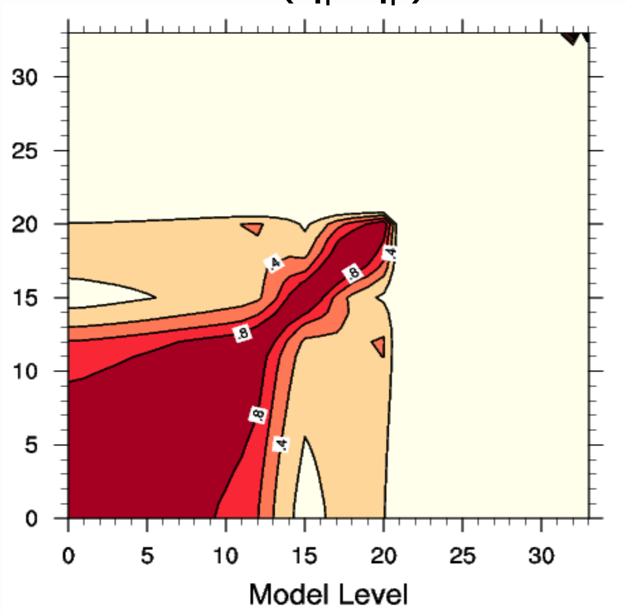
(G : Gaussian blur)

2. Compute multivariate covariances for both classical variables and hydrometeors for each term of the background perturbations

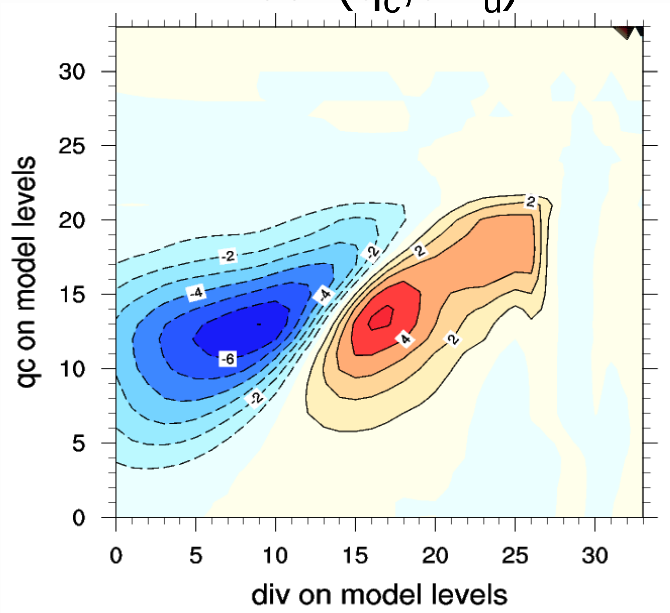
B modelling for hydrometeors

- Multivariate covariances have been computed for WRF by Michel et al. (2011) and for AROME (Martinet et al., 2013)

$cor(q_r, q_r)$



$cov(q_c, div_u)$

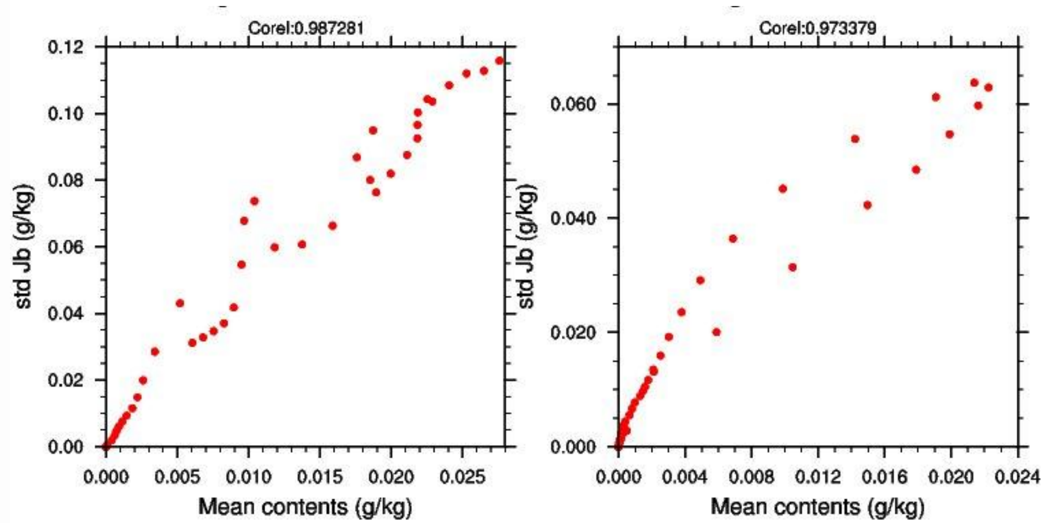


- Vertical correlations strongly linked to the cloud features sampled by the ensemble
- Strong coupling between q_u, T_u and divergence
- Shorter correlation lengths than « classical » variables

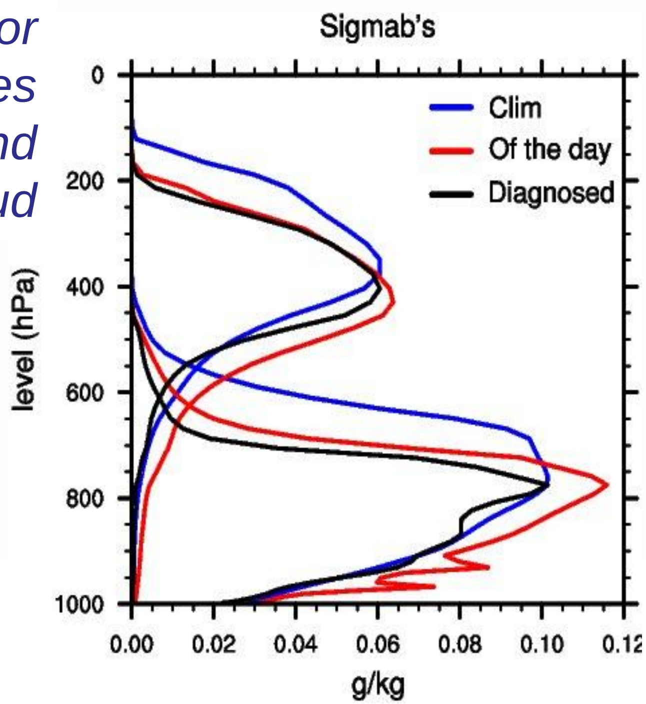
B modelling for hydrometeors

Flow-dependent vertical covariances :
Use of mean contents to distort vertically climatological values

Mean contents vs. error std dev. "of the day" for rain (left) and ice cld (right)



Error variances for rain and ice cloud



- Used in a 1DVar for IASI cloudy radiances (Martinet et al. 2013)
- Ideally, full 3D covariances could be calibrated using daily ensemble

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B deduced from ensemble

In ensemble based methods, flow dependency of forecast errors is provided (entirely or partially) by ensemble perturbations $\mathbf{e}_k = \mathbf{x}^b{}_k - \langle \mathbf{x}^b \rangle$:

$$\mathbf{P_e} = \frac{1}{N-1} \sum_{k=1}^{N} \mathbf{e}_k \mathbf{e}_k^T$$

To avoid distant spurious correlations, to reduce the sampling noise and to increase the rank, covariance localization is applied

$$\mathbf{B_e} = \mathbf{P_e} \circ \mathbf{C}$$

(Houtekamer and Mitchell (2001))

Where $\mathbf{C}$ is a correlation matrix defining horizontal and vertical localization via series of transforms

$\mathbf{C}$ is required to improve the properties of $\mathbf{P_e}$ and can be much simpler than $\mathbf{B_C}$, but should be modeled in a compact way for computational efficiency

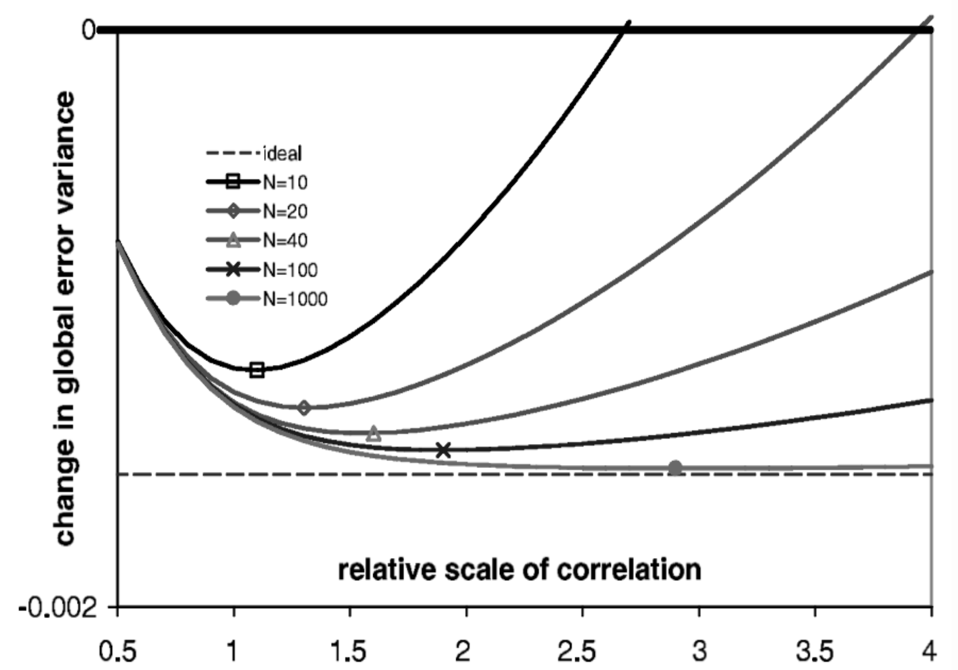
B deduced from ensemble

Ensemble covariances localization

Gaussian shaped-like correlation functions (e.g Gaspari and Cohn, 1999) is commonly used in association with a Shur product

Very empirical and often sub-optimal because correlation lengths depend on:

- number of samples
- resolution and model error
- observation network
- scales of the different physical processes



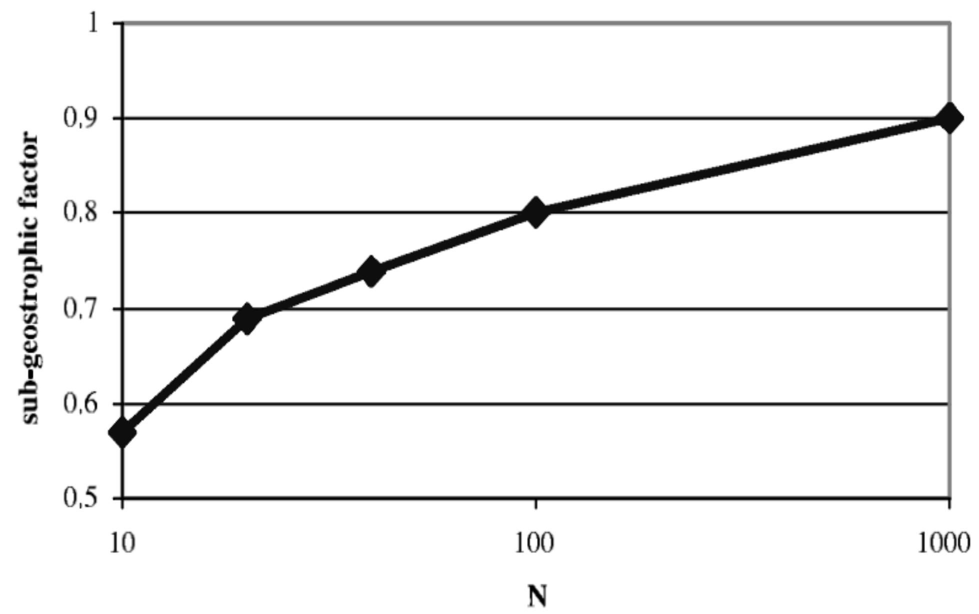
Global error variance vs. relative scale of correlation for different ensemble sizes (Lorenc, 2003)

$\Rightarrow$ At convective scale, these features are even more pronounced!

B deduced from ensemble

Localization causes imbalances

- Balances are directly inherited from the ensemble covariance.
- But, when vertical or horizontal spatial gradients occur, localization implies imbalances



"Sub-geostrophic factor" for different optimal correlation scales associated with different ensemble sizes (Lorenc, 2003)

To alleviate this problem, Clayton et al. (2012) impose balance <u>after</u> localization

Initialization or IAU could also be used, but with a detrimental effect on precipitation forecasts

Towards hybrid methods

	B_c in deterministic VAR	B_e in En-KF like methods
pros	• Balanced analyses • Smoothness and high rank • Stability of the VAR	• Flow dependency, incl. balances • Easy to compute
cons	• Static variances • Homogeneous and isotropic correlations • Sub-optimal in high impact weather situations	• Rank deficient • Ensemble spread • Sampling noise issues • Empirical localization that causes imbalance • Computational cost of the ensemble

Towards hybrid methods

New methods try to merge benefits of the 2 approaches :

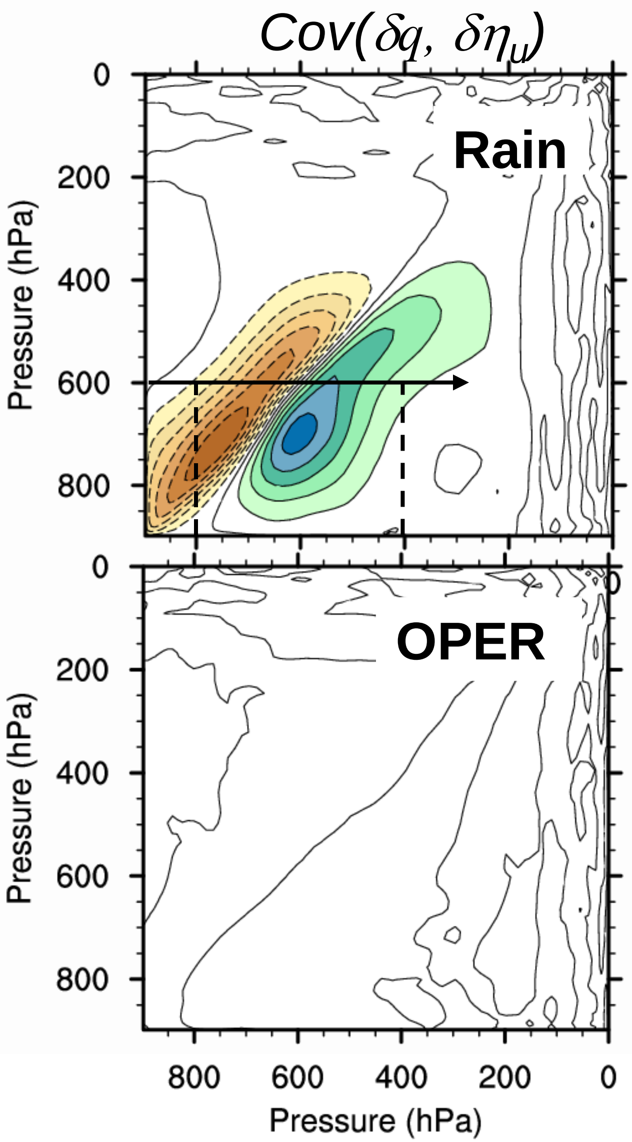
- Some flow dependency can be added in $\mathbf{B_c}^{1/2}$ by estimating its spatial parameters from ensembles (e.g EDA: oper at global scale at MF (Raynaud et al. (2009), Varella et al. (2011)) and ECMWF (Bonavita (2012))

- EDA can also be used to compute $\mathbf{B_c}$ “of the day” for the entire domain (Brousseau et al. 2011) or for specific areas (heterogeneous 3DVar, Montmerle and Berre 2010)

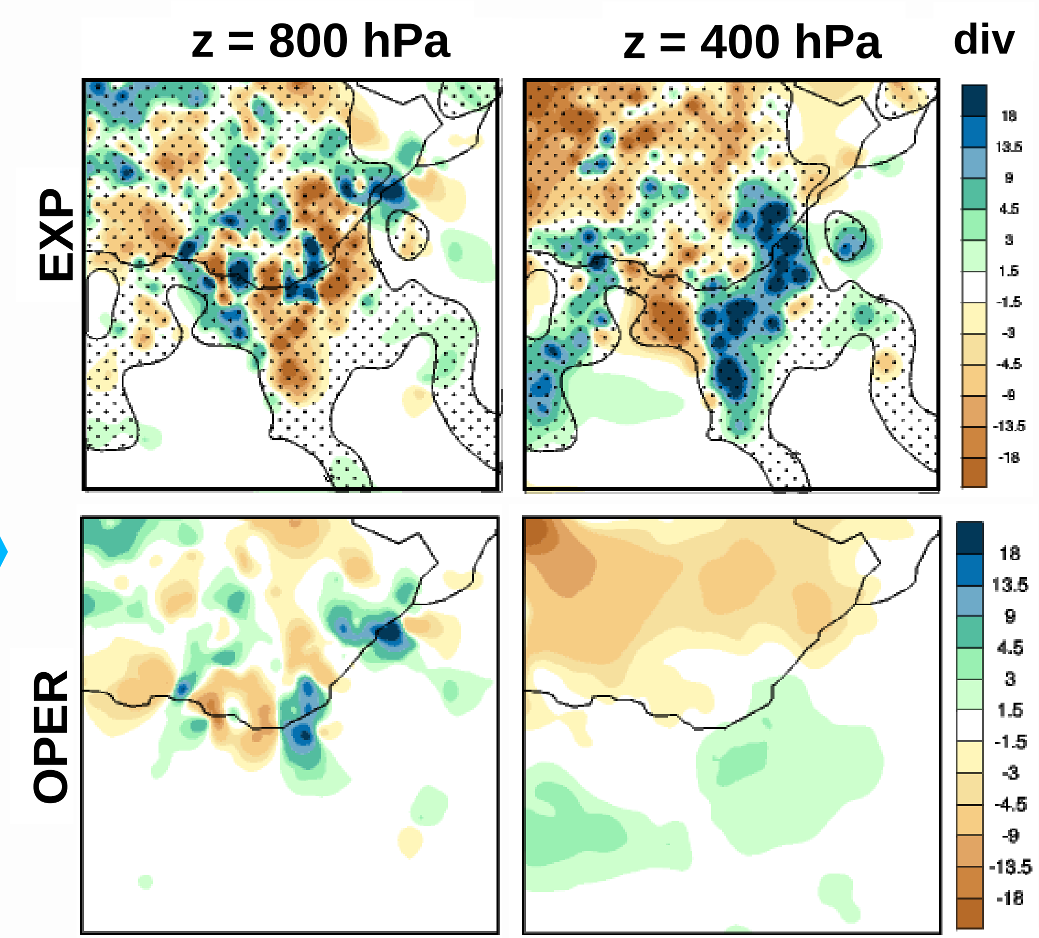
$$\delta x = \mathbf{B}^{1/2}\chi = \begin{pmatrix} \mathbf{F}_1^{1/2}\mathbf{B}_1^{1/2} & \mathbf{F}_2^{1/2}\mathbf{B}_2^{1/2} \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$

Where $\mathbf{F}_1$ and $\mathbf{F}_2$ define the geographical areas where $\mathbf{B}_1$ and $\mathbf{B}_2$ are applied (e.g rainy/clear areas)

Towards hybrid methods



Vertical cross covariances



Divergence increments

(Montmerle MWR 2012)

Towards hybrid methods

The increasingly popular **En-VAR methods** use the full ensemble covariances in an additional term of the variational cost function (Hamill and Snyder (2000); Lorenc (2003); Buehner (2005))

$$\mathbf{P} = \beta_c^2 \mathbf{B}_c + \beta_e^2 \mathbf{L} \circ \mathbf{X}\mathbf{X}^T$$

(where $\mathbf{X}$ is the normalized perturbation matrix and $\mathbf{L}$ contains the localization functions)

•En-VAR merges the advantages of both approaches in a flexible way

•4D-EnVAR is very interesting at convective scale since it avoids the computation of the model's TL/AD

⇒ In all these methods, filtering of covariance parameters from ensembles still is a key point

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Optimal filtering of forecast error parameters from ensembles

Linear filtering of variances:
(Benjamin Ménétrier PhD thesis)

$$\hat{\mathbf{v}} = F\,\tilde{\mathbf{v}} + f = \tilde{\mathbf{v}}^* + \hat{\mathbf{v}}^e$$

Filtered Signal	Gain	raw signal	Offset	True signal	Filtering error

Localization = "Shur linear filtering" of covariances without offset :

$$\widehat{\mathbf{B}} = \mathbf{L} \circ \widetilde{\mathbf{B}}$$

Idea : find an optimality criteria that only uses the input and the output of the filter

Approach : merging theories of the optimal linear filtering and of the centered moments estimation

Methodology : compute iteratively the filtered signal from updated filtering length L_f until the optimality criteria is reached

Optimal filtering of forecast error parameters from ensembles

If the sampling error is supposed unbiased and uncorrelated with the raw signal, a general optimality criteria *C = 0* can be found.

For variances :

$$\mathcal{C}_i = \mathbb{E}\left[\widetilde{v}_i^2\right] - \frac{N(N-2)(N-3)}{(N-1)(N^2-3N+3)}\mathbb{E}\left[\widetilde{v}_i\widehat{v}_i\right] - \frac{N^2}{(N-1)(N^2-3N+3)}\mathbb{E}\left[\widetilde{\xi}_i\right]$$

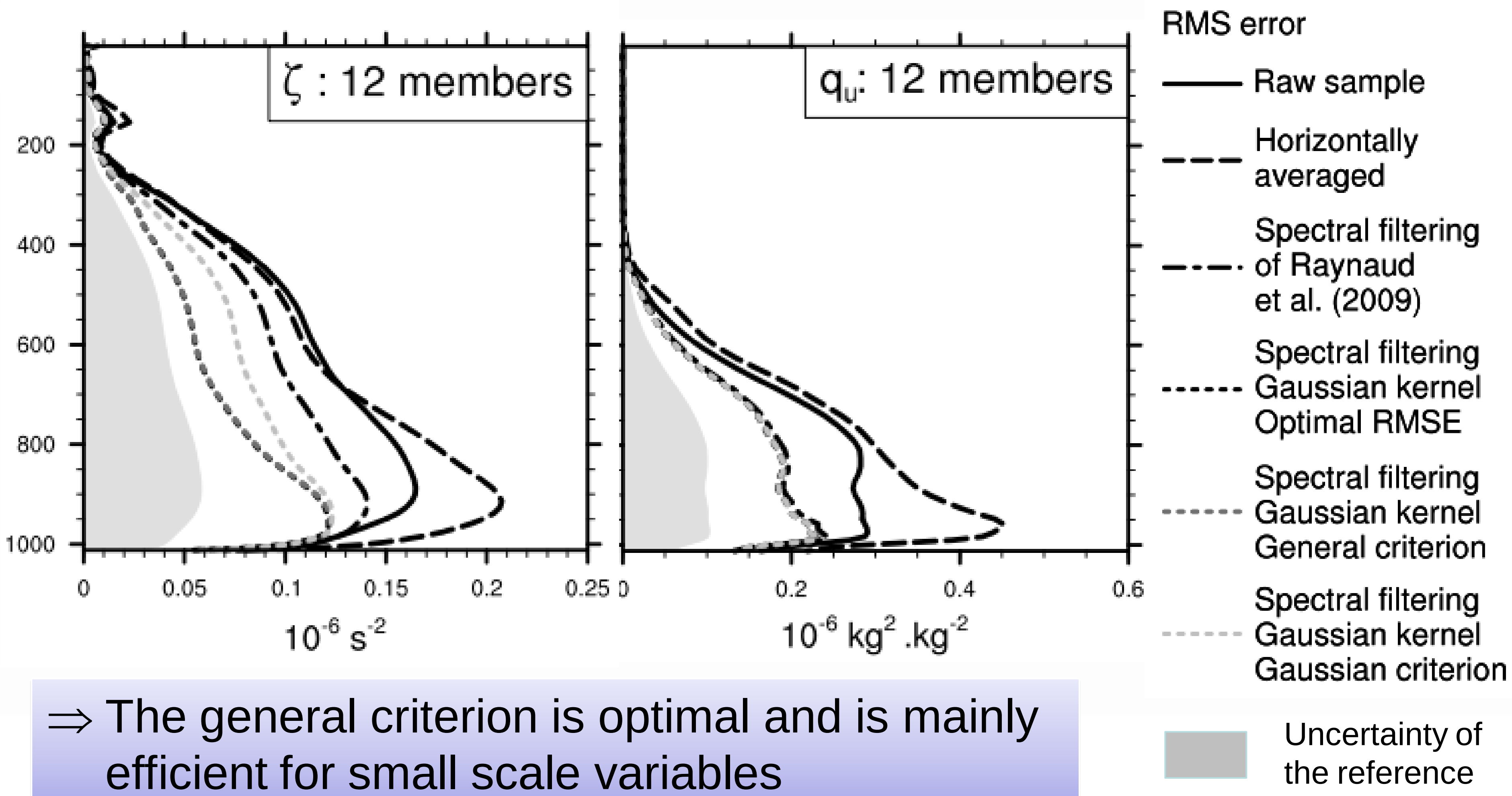
Where ξ is the univariate 4th order moment of the raw signal

If the pdf of the ensemble is Gaussian and if $\mathbb{E}$ is approximated by a spatial average μ :

$$\overline{\mathcal{C}}^G = \mu\left(\widetilde{\mathbf{v}}^2\right) - \frac{N+1}{N-1}\mu\left(\widetilde{\mathbf{v}}\circ\widehat{\mathbf{v}}\right)$$

Optimal filtering of forecast error parameters from ensembles

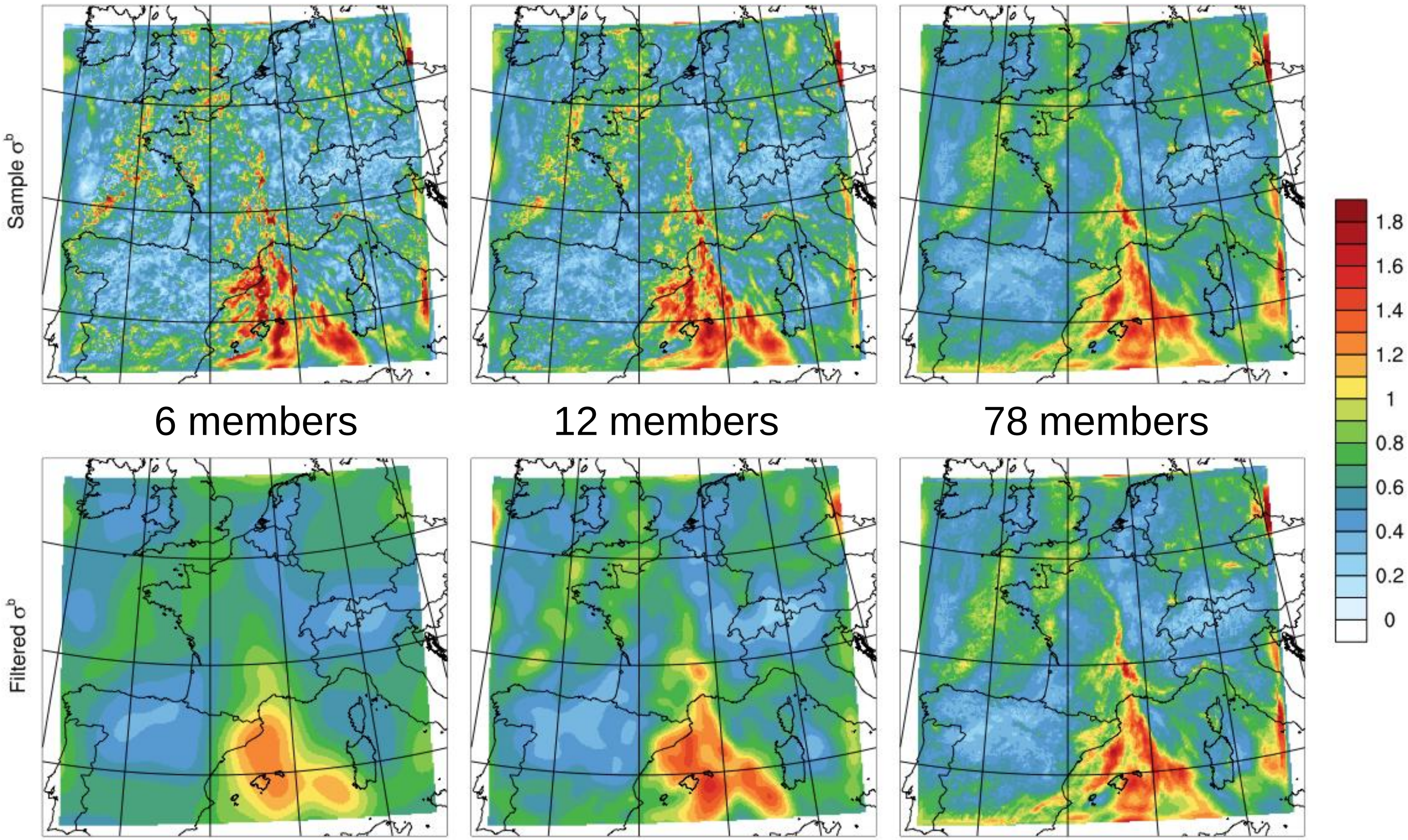
RMS errors of variances against a 78 members AROME reference for vorticity and specific humidity



⇒ The general criterion is optimal and is mainly efficient for small scale variables

Optimal filtering of forecast error parameters from ensembles

Raw (top) and filtered (bottom) variances for q at 950 hPa



L_f decreases with number of members (and varies with height)

Optimal filtering of forecast error parameters from ensembles

Schur filtering

With the same hypothesis of optimal linear filtering and supposing that variances are optimally filtered, the optimal localization length-scale L_{ij} can be computed :

$$L_{ij} = \frac{(N-1)(N^2-3N+3)}{N^3-5N^2+7N-2} - \frac{N^2}{N^3-5N^2+7N-2}\frac{\mathbb{E}\left[\widetilde{\Xi}_{ijij}\right]}{\mathbb{E}\left[\widetilde{B}_{ij}^2\right]} + \frac{N-2}{N^3-5N^2+7N-2}\frac{\mathbb{E}\left[\widetilde{v}_i\widehat{v}_j\right]}{\mathbb{E}\left[\widetilde{B}_{ij}^2\right]}$$

If the sample pdf is Gaussian:
$$L_{ij}^G = \frac{N-1}{N} - \frac{1}{N}\frac{\mathbb{E}\left[\widetilde{v}_i\widehat{v}_j\right]}{\mathbb{E}\left[\widetilde{B}_{ij}^2\right]}$$

For homogeneous and isotropic localization functions, $\mathbb{E}$ is approximated through spatial and angular averages.

Sample correlations can also be used : $\widetilde{C}_{ij} = \widetilde{B}_{ij}/\sqrt{\widetilde{v}_i\widetilde{v}_j}$

$$L_{ij}^{GC} = \frac{(N-1)}{(N+1)(N-2)}\left((N-1) - \frac{1}{\mathbb{E}\left[\widetilde{C}_{ij}^2\right]}\right)$$

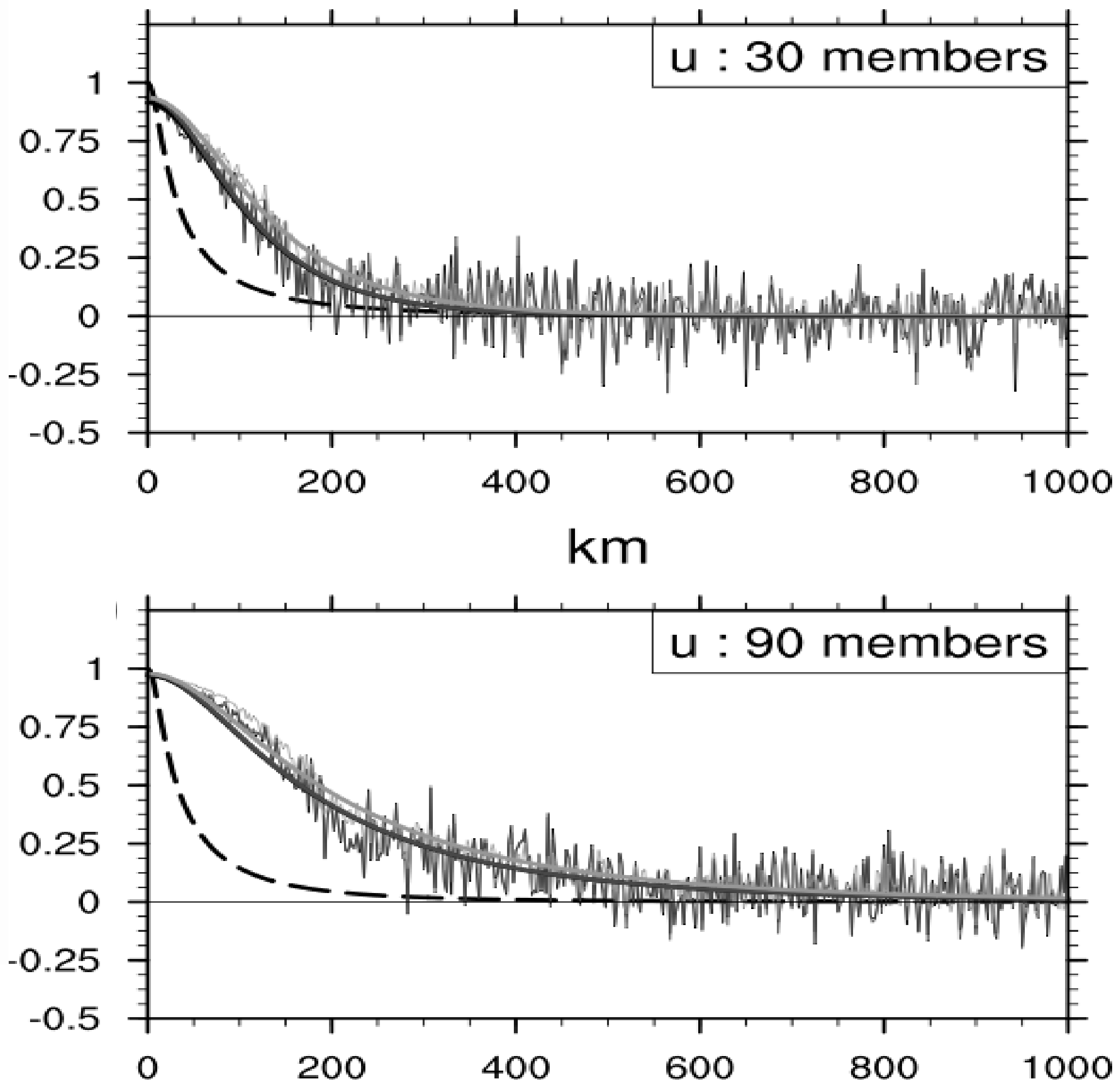
Optimal filtering of forecast error parameters from ensembles

Localization functions for zonal wind at 950 hPa for AROME :

Diagnosed

Fitted

Fitted correlations



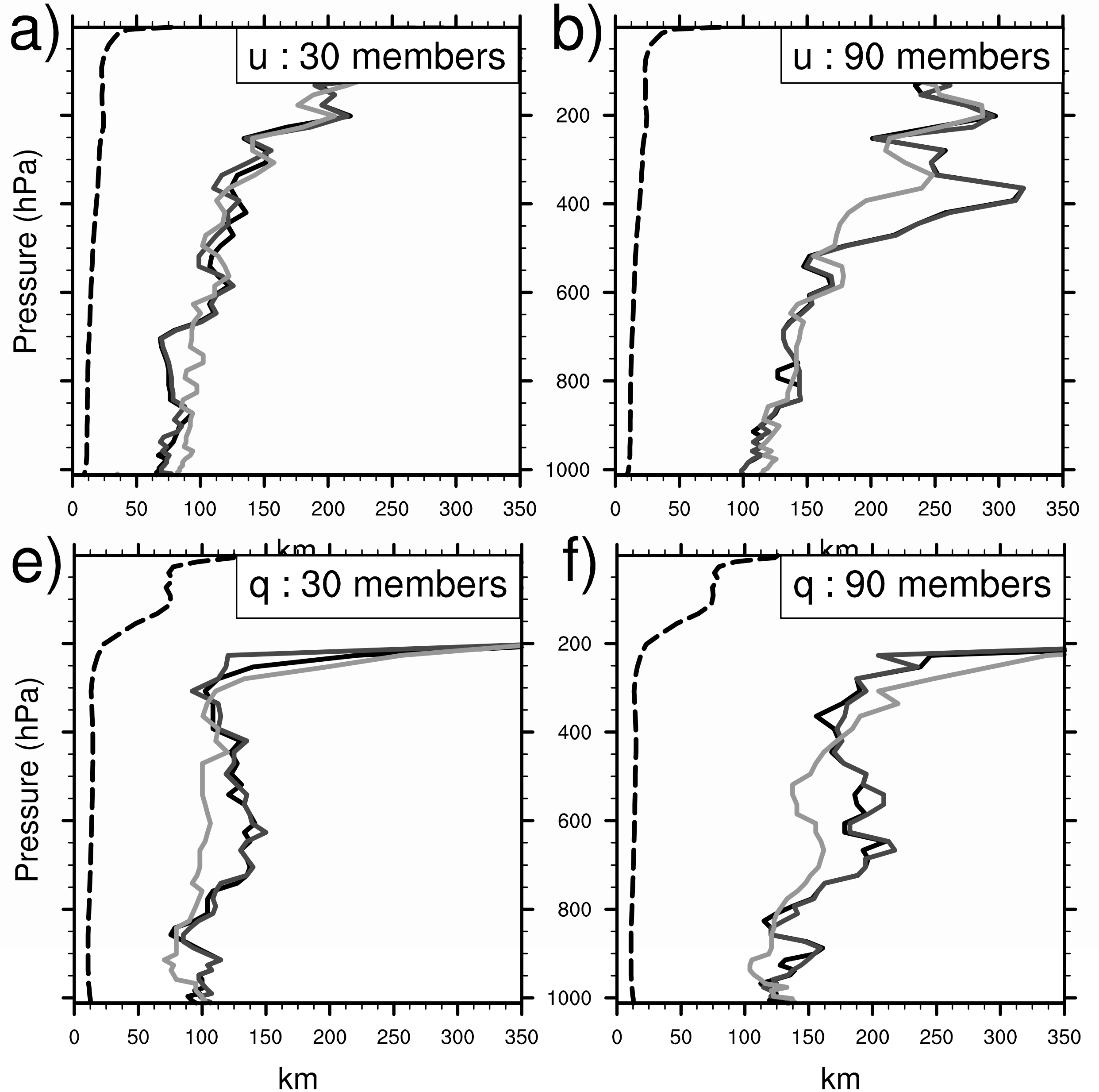
Localization length-scale increases with number of members:
⇒ Long distance correlations are more trusted

Optimal filtering of forecast error parameters from ensembles

Vertical variations of fitted localization functions length-scales for AROME, computed using different formulations.

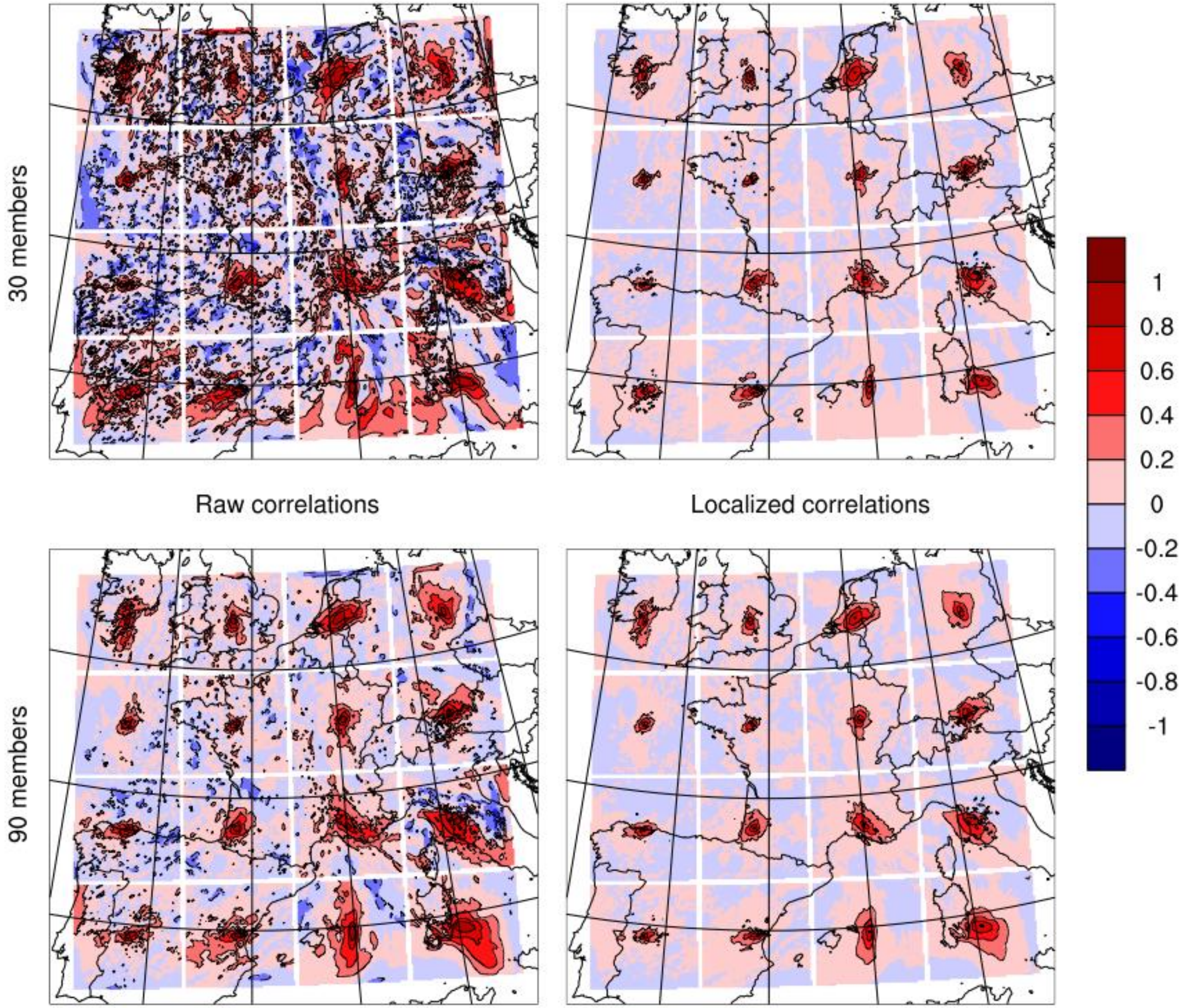
(fitted correlation function length-scales are plotted in dashed lines)

⇒ Important vertical variations for wind (and T)



Optimal filtering of forecast error parameters from ensembles

Raw and localized correlations at 16 different locations for 30 and 90 members (q at 950 hPa)



Outlines

- Specific features compared to global scale
- **B** modelling
    - Climatological formulation
    - Adding some flow dependencies
    - Considering hydrometeors
- **B** totally or partly deduced from ensembles
    - principles
    - role of localization
- Towards optimal filtering of forecast error parameters from ensembles
- **Conclusions**

Conclusions

- Forecast errors at convective scale display features linked to the explicit convection, to diabatic processes, to the type of surface, to the coupling files, to the specific observation network (e.g radars)
- Significant differences have been shown with global scale
- Operational formulations of $\mathbf{B_c}$ are clearly sub-optimal, especially for LAM in regions characterized by high impact weather (e.g clouds and precipitations)
- Flow dependencies can be provided from ensembles, whether in $\mathbf{B_c}$ or using an En-VAR formulation

Conclusions

Operationally, the set up of an ensemble still is difficult because :

- need of perturbed LBCs
- the computational cost
- the estimation and the representation of model error
- sampling noise is severe, especially at CS

$\Rightarrow$ Cheaper ensembles in the limit of the "grey zone" (providing that explicit convection is activated) could be an option

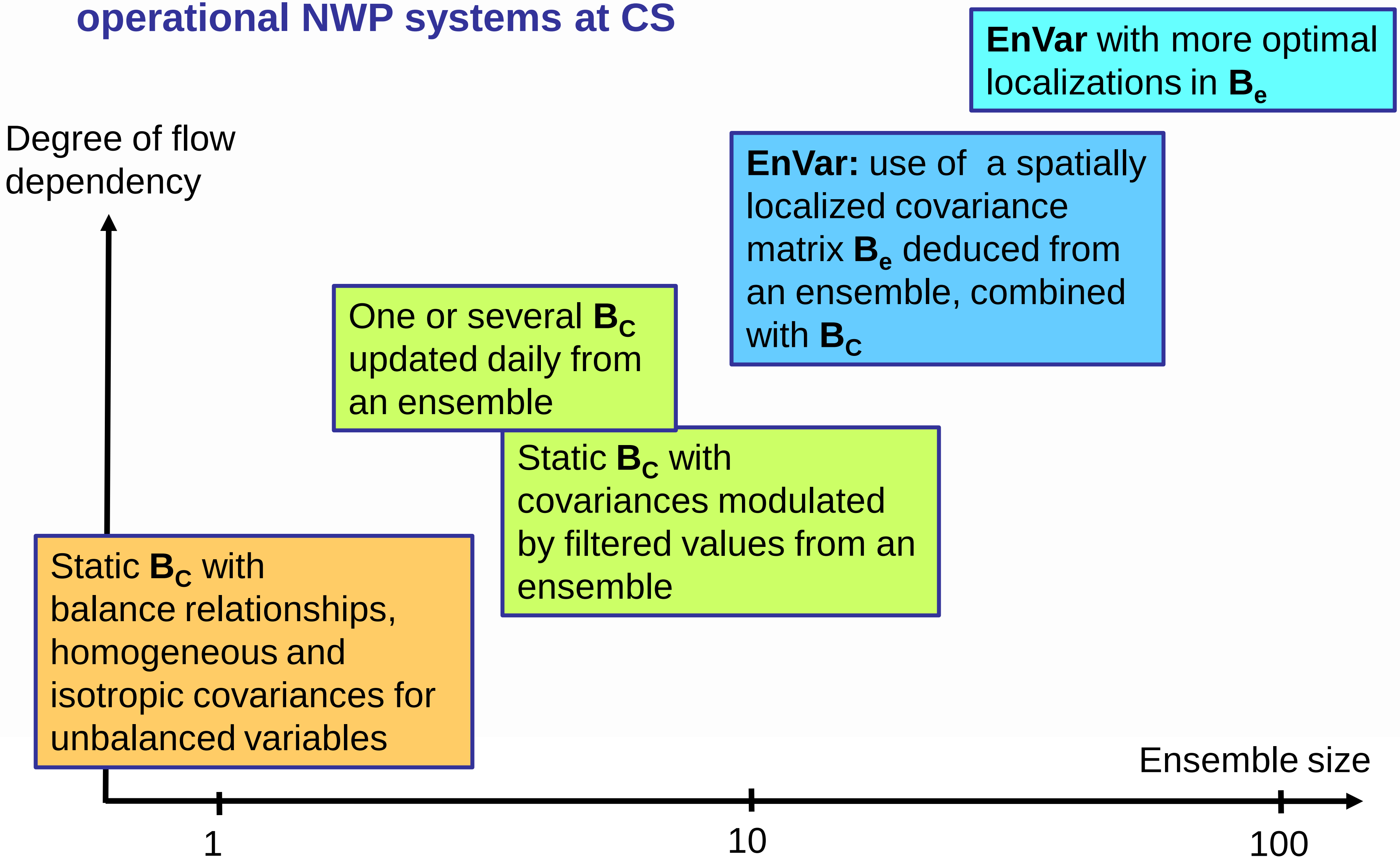
$\Rightarrow$ An optimal filtering of forecast error parameters is essential

$\Rightarrow$ Such filtering depends strongly on the ensemble size, on the altitude and on the variable

$\Rightarrow$ Results can eventually be validated and tuned using innovation-based diagnostics

Conclusions

Possible evolution of B in operational NWP systems at CS



Thank you for your attention !

References

Bannister, R. N., 2008: A review of forecast error covariance statistics in atmospheric variational data assimilation. I: Characteristics and measurements of forecast error covariances. Quart. J. Roy. Meteor. Soc., 134, 1951–1970.

Berre L., 2000 : Estimation of synoptic and mesoscale forecast error covariances in a limited area model, Mon. Wea. Rev., 128, 644-667.

Berre L, Stefanescu SE, Belo-Pereira M. 2006. The representation of a the analysis effect in three error simulation techniques. Tellus 58A: 196–209.

Bishop, C. H., and D. Hodyss, 2009b: Ensemble covariances adaptively localized with ECO-RAP. Part 2: A strategy for the atmosphere. Tellus, 61A, 97–111.

Brousseau, P., L. Berre, F. Bouttier, and G. Desroziers, 2012: Flow-dependent background-error covariances for a convective scale data assimilation system. Quart. J. Roy. Meteor. Soc., 138, 310–322

Buehner, M., 2005: Ensemble-derived stationary and flow-dependent background-error covariances: Evaluation in a quasi-operational

NWP setting. Quart. J. Roy. Meteor. Soc., 131, 1013–1043.

Buehner, M, and M. Charron, 2007: Spectral and spatial localization of background error correlations for data assimilation. Quart. J. Roy. Meteor. Soc., 133, 615–630.

Campbell, C., Bishop, C. and Hodyss D: 2010: Vertical Covariance Localization for Satellite Radiances in Ensemble Kalman Filters. MWR, 138, 282-290.

Caron, J.-F., and L. Fillion, 2010: An examination of background error correlations between mass and rotational wind over precipitation regions. Mon. Wea. Rev., 138, 563–578.

Clayton, A. C. Lorenc and D. M. Barker, 2012: Operational implementation of a hybrid ensemble/4D-Var global data assimilation system at the Met Office. QJRMS.

Deckmyn, A., and L. Berre, 2005 : A wavelet approach to representing background error covariances in a limited area model. Mon. Wea. Rev., 133, 1279-1294.

Derber, J., and F. Bouttier, 1999: A reformulation of the background error covariance in the ECMWF global data assimilation system. Tellus, 51A, 195–221.

Desroziers G., L. Berre, O. Pannekoucke, S. Stefanescu, P. Brousseau, L. Auger, B. Chapnik, and L. Raynaud, 2008. Flow-dependent error covariances from variational assimilation ensembles on global and regional domains. HIRLAM Techn. Report, 68 :5–22, 2008. 4.1.3

Ehrendorfer M. 2007. A review of issues in ensemble-based Kalman filtering Meteorol. Z. 16: 795–818.

Fisher, M., 2003 : Background error covariance modelling. Proceedings of the ECMWF seminar on recent developments in data assimilation for atmosphere and ocean, 45-63.

Gaspari, G., and S. E. Cohn, 1999: Construction of correlation functions in two and three dimensions. Quart. J. Roy. Meteor. Soc., 125, 723–757.

Houtekamer, P. L., Lefaivre, L., Derome, J., Ritchie, H. and Mitchell, H. L. 1996. A system simulation approach to ensemble prediction. Mon. Weather Rev. 124, 1225–1242.

Houtekamer, P. L. and Mitchell, H. L., 2011:A sequential ensemble Kalman lter for atmospheric data assimilation. Mon. Weather Rev., 129, 123–137

Lorenc AC. 2003. The potential of the ensemble Kalman filter for NWP - a comparison with 4D-Var. Q. J. R. Meteorol. Soc. 129: 3183–3203.

Martinet P., N. Fourrié, V. Guidard, F. Rabier, T. Montmerle, and P. Brunel, 2013 :Towards the use of microphysical variables for the assimilation of cloud-affected infrared radiances. Quart. J. Roy. Meteor. Soc.

Ménétrier, B., and T. Montmerle, 2011: Heterogeneous background error covariances for the analysis and forecast of fog events. Quart. J. Roy. Meteor. Soc., 137, 2004–2013.

Ménétrier B, Montmerle T., Berre L. and Michel Y., 2013: Estimation and diagnosis of heterogeneous flow-dependent background error covariances at convective scale using either large or small ensembles. QJRMS, in press.

Michel, Y., Auligné T. and T. Montmerle, 2011 : Diagnosis of heterogeneous convective-scale Background Error Covariances with the inclusion of hydrometeor variables. Mon.Wea Rev., 138(1), 101–120.

Michel Y. 2012. Estimating deformations of random processes for correlation modelling: methodology and the one-dimensional case. Quarterly Journal of the Royal Meteorological Society 139: 771–783.

Montmerle T. and L. Berre, 2010: Diagnosis and formulation of heterogeneous background error covariances at mesoscale. Quart. J. Roy. Meteor. Soc., 136, 1408–1420.

·Montmerle T., 2012: Optimization of the assimilation of radar data at convective scale using specific background error covariances in precipitations. Mon. Wea Rev., 140, 3495-3506.

Pagé C, Fillion L, Zwack P. 2007. Diagnosing summertime mesoscale vertical motion: implications for atmospheric data assimilation. Mon. Weather Rev. 135: 2076–2094.

Purser RJ, WuWS, Parrish DF, Roberts NM. 2003. Numerical aspects of the application of recursive filters to variational analysis. Part I: Spatially homogeneous and isotropic Gaussian covariances. Mon. Weather Rev. 131: 1524–1535.

Varella H, Berre L, Desroziers G. 2011. Diagnostic and impact studies of a wavelet formulation of background-error correlations in a global model. Q. J. R. Meteorol. Soc. 137: 1369–1379.

Vetra-Carvalho S., M. Dixon, S. Migliorini, N. K. Nichols and S. P. Ballard, 2012: Breakdown of hydrostatic balance at convective scales in the forecast errors in the Met Office Unified Model. QJRMS, 138: 1709–1720

Weaver AT, Mirouze I. 2012. On the diffusion equation and its application to isotropic and anisotropic correlation modelling in variational assimilation. Quarterly Journal of the Royal Meteorological Society 139: 242–260.

Zhang,F, M. Zhang, and J. A. Hansen, 2009b: Coupling ensemble Kalman filter with four-dimensional variational data assimilation. Adv. Atmos. Sci., 26, 1–8.